Mathematics — Solved Paper 2015

SECTION A (40 Marks)

*(Answer **all** questions from this Section)*

***Question* 1 :**

(a) A shopkeeper bought an article for ₹ 3,450. He marks the price of the article 16% above the cost price. The rate of sales tax charged on the article is 10%. Find the :

(i) marked price of the article.

(ii) price paid by a customer who buys the article. [3]

Solution :

(i) **For shopkeeper :**

C.P. of the article = ₹ 3,450

and, **M.P.** of the article = ₹ 3,450 + 16% of ₹ 3,450

= ₹ 3,450 + ₹ 552 = **₹ 4,002** **Ans.**

(ii) $\because$ Sales tax = 10% of ₹ 4,002 = ₹ 400.20

$\therefore$ **Customer paid** = ₹ 4,002 + ₹ 400.20 = **₹ 4,402.20** **Ans.**

(b) Solve the following inequation and write the solution set :

$13x - 5 < 15x + 4 < 7x + 12,\ x \in R$

Represent the solution on a real number line. [3]

Solution :

$$13x - 5 < 15x + 4 < 7x + 12$$

$\Rightarrow \quad 13x - 5 < 15x + 4 \quad$ and $\quad 15x + 4 < 7x + 12$

$\Rightarrow \quad -2x < 9 \quad$ and $\quad 8x < 8$

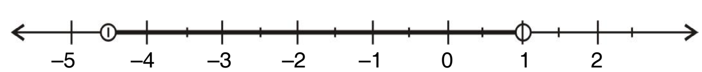
$\Rightarrow \quad 2x > -9 \quad$ and $\quad x < 1$

$\Rightarrow \quad x > -4{\cdot}5$

Combining we get : $-4{\cdot}5 < x < 1$

$\therefore$ **Solution set = $\{x : x \in R$ and $-4{\cdot}5 < x < 1\}$** **Ans.**

Solution on a real number line :



Ans.

(c) Without using trigonometric tables evaluate :

$$\mathbf{\frac{\sin 65^\circ}{\cos 25^\circ} + \frac{\cos 32^\circ}{\sin 58^\circ} - \sin 28^\circ.\ \sec 62^\circ + \operatorname{cosec}^2 30^\circ.}$$ **[4]**

Solution :

Since, $\sin 65^\circ = \sin (90^\circ - 25^\circ) = \cos 25^\circ$

$\sin 58^\circ = \sin (90^\circ - 32^\circ) = \cos 32^\circ$

and, $\sec 62^\circ = \sec (90^\circ - 28^\circ) = \operatorname{cosec} 28^\circ = \dfrac{1}{\sin 28^\circ}$

$$\therefore \quad \mathbf{\frac{\sin 65^\circ}{\cos 25^\circ} + \frac{\cos 32^\circ}{\sin 58^\circ} - \sin 28^\circ.\ \sec 62^\circ + \operatorname{cosec}^2 30^\circ}$$

$$= \frac{\cos 25°}{\cos 25°} + \frac{\cos 32°}{\cos 32°} - \sin 28°. \frac{1}{\sin 28°} + (2)^2$$

$= 1 + 1 - 1 + 4 = \mathbf{5}$ **Ans.**

***Question* 2 :**

(a) If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x and y when $A^2 = B$. **[3]**

Solution :

$$A^2 = A \times A$$

$$= \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9+0 & 3x+x \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$$

Since, $A^2 = B \quad \Rightarrow \quad \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$\Rightarrow \quad 4x = 16$ and $1 = -y$ *i.e.* $\mathbf{x = 4}$ and $\mathbf{y = -1}$ **Ans.**

(b) The present population of a town is 2,00,000. Its population increases by 10% in the first year and 15% in the second year. Find the population of the town at the end of the two years. **[3]**

Solution :

Population at the end of two years

$$= 2{,}00{,}000 \left(1+\frac{10}{100}\right)\left(1+\frac{15}{100}\right) = \mathbf{2{,}53{,}000}$$ **Ans.**

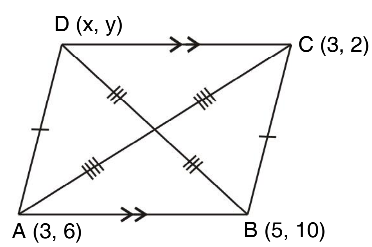
(c) Three vertices of a parallelogram ABCD taken in order are A (3, 6), B (5, 10) and C (3, 2), find :

(i) the coordinates of the fourth vertex D.

(ii) length of diagonal BD.

(iii) equation of side AB of the parallelogram ABCD. **[4]**

Solution :



(i) Let D = (x, y).

Since, diagonals of a parallelogram bisect each other.

$\therefore$ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{3+3}{2}, \frac{6+2}{2}\right) = \left(\frac{5+x}{2}, \frac{10+y}{2}\right)$$

$\Rightarrow x = 1$ and $y = -2$ *i.e.* **D = (1, –2)** **Ans.**

(ii) $\because$ B = (5, 10) and D = (1, –2)

$$\therefore \mathbf{BD} = \sqrt{(5-1)^2 + (10+2)^2}$$

$$= \sqrt{16+144} = \sqrt{160} = \sqrt{16 \times 10} = \mathbf{4\sqrt{10}}$$ **Ans.**

(iii) $\because$ $\quad A = (3, 6) = (x_1, y_1)$ and $B = (5, 10) = (x_2, y_2)$

$\therefore$ Slope of AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{10-6}{5-3} = \dfrac{4}{2} = 2$

$\because$ Slope $(m) = 2$ and $A = (3, 6) = (x_1, y_1)$

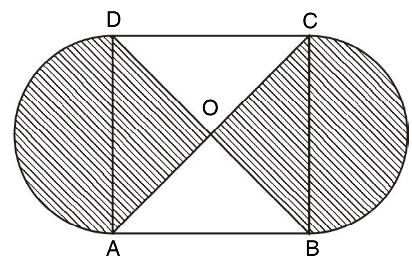
Equation of AB :

$y - y_1 = m(x - x_1) \Rightarrow y - 6 = 2(x - 3)$

$\Rightarrow y - 6 = 2x - 6$ *i.e.* $\boldsymbol{y = 2x}$ **Ans.**

***Question* 3 :**

(a) In the given figure, ABCD is a square of side 21 cm. AC and BD are two diagonals of the square. Two semi circles are drawn with AD and BC as diameters. Find the area of the shaded region. (Take $\pi = \dfrac{22}{7}$). [3]



Solution :

Area of (ΔAOD + ΔBOC)

$= \dfrac{1}{2} \times$ area of square ABCD

$= \dfrac{1}{2} \times (21)^2$ sq. cm $= 220{\cdot}5$ sq. cm

Area of two semi-circles

= area of a circle with diameter 21 cm

$= \pi r^2$

$= \dfrac{22}{7} \times \dfrac{21}{2} \times \dfrac{21}{2}$ sq. cm

$= 346{\cdot}5$ sq. cm

$\therefore$ **Area of the shaded region**

$= 220{\cdot}5$ sq. cm $+ 346{\cdot}5$ sq. cm $=$ **567 sq. cm** **Ans.**

(b) The marks obtained by 30 students in a class assessment of 5 marks is given below :

Marks	**0**	**1**	**2**	**3**	**4**	**5**
No. of students	**1**	**3**	**6**	**10**	**5**	**5**

Calculate the mean, median and mode of the above distribution. [3]

Solution :

Marks *(x)*	**No. of students** *(f)*	*fx*	***Comulative frequency***
0	1	0	1
1	3	3	4
2	6	12	10
3	10	30	20
4	5	20	25
5	5	25	30
	$n = 30$	$\Sigma fx = 90$	

For median

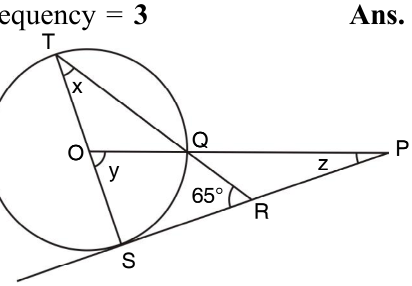
$$\textbf{Mean} = \frac{\sum fx}{n} = \frac{90}{30} = \mathbf{3} \qquad \textbf{Ans.}$$

$$\textbf{Median} = \frac{1}{2}\left[\left(\frac{30}{2}\right)^{th} \text{term} + \left(\frac{30}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[15^{th} \text{ term} + 16^{th} \text{ term}\right] = \frac{1}{2}(3 + 3) = \mathbf{3} \qquad \textbf{Ans.}$$

Mode = The number (marks) with highest frequency = **3** **Ans.**

(c) In the figure given alongside, O is the centre of the circle and SP is a tangent. If ∠SRT = 65°, find the values of *x*, *y* and *z*. **[4]**



Solution :

$\because$ Angle between radius OS and tangent SP = 90°

$\Rightarrow$ ∠OSP = 90° = ∠TSP

In ΔTSR, x + 90° + 65° = 180° $\Rightarrow$ $\boldsymbol{x}$ **= 25°** **Ans.**

In ΔTOQ, OT = OQ = radius

$\Rightarrow$ ∠OQT = x = 25°

$\therefore$ Exterior angle $y = x$ + ∠OQT *i.e.* $\boldsymbol{y}$ = 25° + 25° = **50°** **Ans.**

> **Alternative method for finding the value of *y* :**
> Minor arc SQ subtends angle y at the centre and angle x at point T on the remaining circumference of the circle. Also, angle at centre is twice the angle at remaining circumference $\Rightarrow y = 2x$ *i.e.* $\boldsymbol{y}$ = 2 × 25° = **50°** **Ans.**

In ΔOSP, y + ∠OSP + z = 180°

$\Rightarrow$ 50° + 90° + z = 180° $\Rightarrow$ $\boldsymbol{z}$ **= 40°** **Ans.**

Question **4 :**

(a) Katrina opened a recurring deposit account with a Nationalised Bank for a period of 2 years. If the bank pays interest at the rate of 6% per annum and the monthly instalment is ₹ 1,000, find the :

(i) interest earned in 2 years.

(ii) matured value. **[3]**

Solution :

(i) $\because$ Monthly instalment (P) = ₹ 1,000

no. of instalments (n) = $2 \times 12 = 24$

and, rate (r) = 6%

$$\therefore \quad \textbf{Interest} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= ₹\, 1{,}000 \times \frac{24 \times 25}{2 \times 12} \times \frac{6}{100} = \textbf{₹ 1,500} \qquad \textbf{Ans.}$$

(ii) $\because$ Total money deposited = $P \times n$

= ₹ 1,000 × 24 = ₹ 24,000

$\therefore$ **Matured value** = Money deposited + Interest

= ₹ 24,000 + ₹ 1,500 = **₹ 25,500** **Ans.**

(b) Find the value of 'K' for which $x = 3$ is a solution of the quadratic equation, $(K + 2)\, x^2 - Kx + 6 = 0$

Thus find the other root of the equation. **[3]**

Solution :

$x = 3$ is a solution of equation $(K + 2)x^2 - Kx + 6 = 0$

$\Rightarrow \quad (K + 2) \times 9 - K \times 3 + 6 = 0$

$\Rightarrow \quad 9K + 18 - 3K + 6 = 0 \quad$ *i.e.* $\quad 6K = -24$ and $\mathbf{K = -4}$ **Ans.**

For K = –4, $\quad (K + 2)x^2 - Kx + 6 = 0$

$\Rightarrow \quad -2x^2 + 4x + 6 = 0 \quad$ *i.e.* $\quad x^2 - 2x - 3 = 0$

$\Rightarrow \quad x^2 - 3x + x - 3 = 0 \quad$ *i.e.* $\quad x(x - 3) + 1(x - 3) = 0$

$\Rightarrow \quad (x - 3)(x + 1) = 0 \quad$ *i.e.* $\quad x - 3 = 0 \quad$ or $\quad x + 1 = 0$

$\Rightarrow \quad x = 3 \quad$ or $\quad x = -1$

Since, $x = 3$ is already given to be one root (solution) of the equation.

$\therefore$ The other root of the equation is $\boldsymbol{x = -1}$. **Ans.**

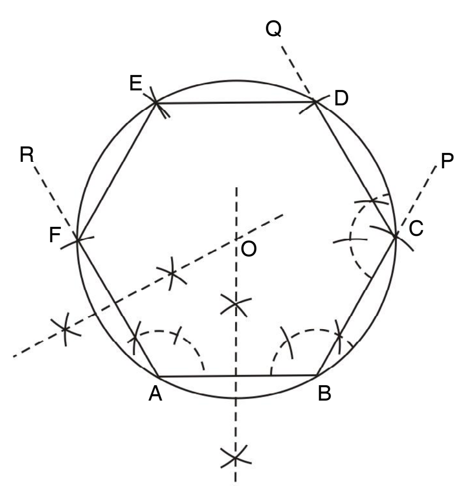
(c) Construct a regular hexagon of side 5 cm. Construct a circle circumscribing the hexagon. All traces of construction must be clearly shown. **[4]**

Solution :

Steps :

1. Draw AB = 5 cm.
2. Construct angle ABP = 120° and from BP cut BC = 5 cm.
3. Construct angle BCQ = 120° and from CQ cut CD = 5 cm.
4. Construct angle BAR = 120° and from AR cut AF = 5 cm.
5. Taking D and F as centres, draw two equal arcs each of radius 5 cm. Let these arcs intersect each other at point E. Join EF and ED.

   ABCDEF is the required regular hexagon of side 5 cm.



Further, draw perpendicular bisectors of any two non-parallel sides of the hexagon. Here, the perpendicular bisectors of sides AB and AF are drawn, which meet each other at point O.

Taking O as centre and OA as radius, draw a circle which will pass through all the vertices of the hexagon ABCDEF and is the required circumcircle.

SECTION B (40 Marks)

*(Answer any **four** questions from this Section)*

***Question* 5 :**

(a) Use a graph paper for this question taking 1 cm = 1 unit along the x axis and the y axis both :

(i) Plot the points A(0, 5), B(2, 5), C(5, 2), D(5, –2), E(2, –5) and F(0, –5).

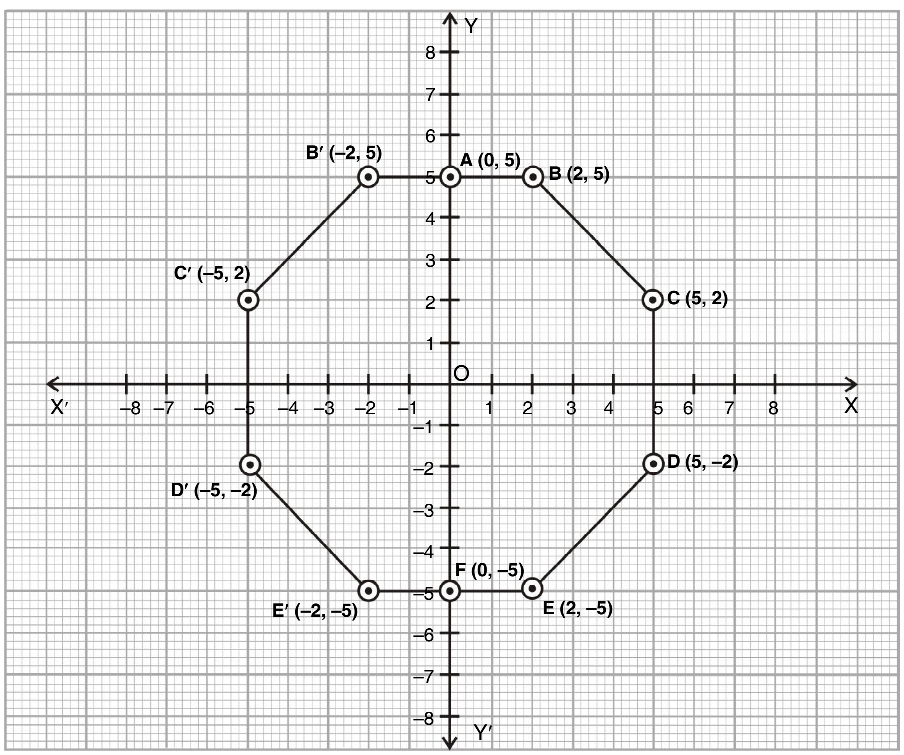
(ii) Reflect the points B, C, D and E on the y-axis and name them respectively as B′, C′, D′ and E′.

(iii) Write the coordinates of B′, C′, D′ and E′.

(iv) Name the figure formed by B C D E E′ D′ C′ B′.

(v) Name a line of symmetry for the figure formed. [5]

Solution :



(iii) **B′** = **(–2, 5)**, **C′** = **(–5, 2)**, **D′** = **(–5, –2)** and **E′** = **(–2, – 5)**. **Ans.**

(iv) **Octagon**. **Ans.**

(v) x-axis *i.e.* $y = 0$ **Ans.**

[Here, for the figure formed, y-axis *i.e.* $x = 0$ may also be taken as line of symmetry].

(b) Virat opened a Saving Bank account in a bank on 16th April 2010. His pass book shows the following entries :

Date	Particulars	Withdrawal (₹)	Deposit (₹)	Balance (₹)
April 16th, 2010	**By cash**	–	**2500**	**2500**
April 28th	**By cheque**	–	**3000**	**5500**
May 9th	**To cheque**	**850**	–	**4650**
May 15th	**By cash**	–	**1600**	**6250**
May 24th	**To cash**	**1000**	–	**5250**
June 4th	**To cash**	**500**	–	**4750**
June 30th	**By cheque**	–	**2400**	**7150**
July 3rd	**By cash**	–	**1800**	**8950**

Calculate the interest Virat earned at the end of 31st July, 2010 at 4% per annum interest. What sum of money will he receive if he closes the account on 1st August, 2010 ? [5]

Solution :

Principal for April = ₹ 00 [Account is opened on 16th April]

Principal for May = ₹ 4,650

Principal for June = ₹ 4,750

Principal for July = ₹ 8,950

Total principal for one month = ₹ 18,350

Interest earned $= \dfrac{\text{PRT}}{100}$

$= \dfrac{₹\,18{,}350 \times 4 \times 1}{100 \times 12} = \textbf{₹ 61.17}$ **Ans.**

Money received on closing the account on 1st August, 2010

= Last balance + Interest earned

= ₹ 8,950 + ₹ 61.17 = **₹ 9,011.17** **Ans.**

***Question* 6 :**

(a) If *a*, *b*, *c* are in continued proportion, prove that
$\boldsymbol{(a + b + c)(a - b + c) = a^2 + b^2 + c^2}$. **[3]**

Solution :

a, b, c are in continued proportion $\Rightarrow b^2 = ac$

Now, $\boldsymbol{(a + b + c)(a - b + c)} = (a + c)^2 - b^2$

$= a^2 + c^2 + 2ac - b^2$

$= a^2 + c^2 + 2b^2 - b^2$ $[\because ac = b^2]$

$= \boldsymbol{a^2 + b^2 + c^2}$ **Hence Proved**

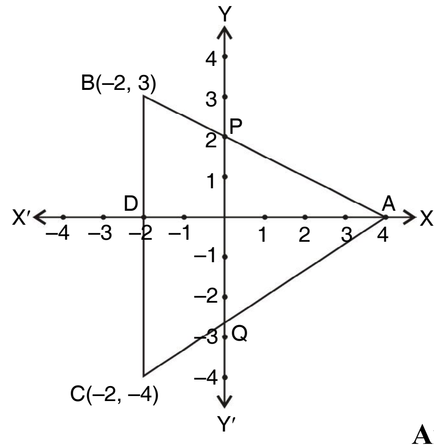
(b) In the figure, given alongside, ABC is a triangle and BC is parallel to the y-axis. AB and AC intersect the y-axis at P and Q respectively.

(i) Write the coordinates of A.

(ii) Find the length of AB and AC.

(iii) Find the ratio in which Q divides AC.

(iv) Find the equation of the line AC. **[4]**



Solution :

(i) **A = (4, 0)** **Ans.**

(ii) $\textbf{AB} = \sqrt{(4+2)^2+(0-3)^2}$ [$\because$ A = (4, 0) and B = (–2, 3)]

$= \sqrt{36+9} = \sqrt{45} = \sqrt{9\times 5} = \mathbf{3\sqrt{5}}$ **unit** **Ans.**

$\textbf{AC} = \sqrt{(4+2)^2+(0+4)^2}$ [$\because$ A = (4, 0) and C = (–2, –4)]

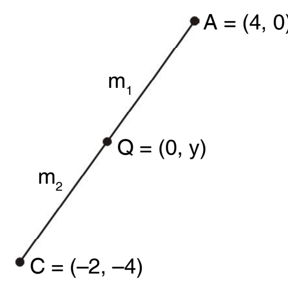
$= \sqrt{36+16} = \sqrt{52} = \sqrt{4\times 13} = \mathbf{2\sqrt{13}}$ **unit** **Ans.**

(iii) Let Q divides AC in the ratio $m_1 : m_2$ *i.e.* AQ : QC = $m_1 : m_2$

$\because \quad \boldsymbol{x} = \dfrac{m_1x_2+m_2x_1}{m_1+m_2}$

$\Rightarrow \quad 0 = \dfrac{m_1\times -2+m_2\times 4}{m_1+m_2}$ *i.e.* $0 = -2m_1 + 4m_2$

$\Rightarrow \quad \dfrac{m_1}{m_2} = 2$ or $\dfrac{2}{1}$ *i.e.* $\boldsymbol{m_1 : m_2 = 2 : 1}$ **Ans.**



(iv) **For AC**

Slope $(m) = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{0+4}{4+2} = \dfrac{2}{3}$ and $(x_1, y_1) = (4, 0)$

$\therefore$ Equation of AC is : $y - y_1 = m(x - x_1)$

$\Rightarrow \quad y + 0 = \dfrac{2}{3}(x - 4)$

$\Rightarrow \quad 3y = 2x - 8$

i.e. $\mathbf{2x - 3y = 8}$ **Ans.**

(c) Calculate the mean of the following distribution :

Class interval	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
Frequency	**8**	**5**	**12**	**35**	**24**	**16**

[3]

Solution :

C.I.	f	Class-mark (x)	fx
0-10	8	5	40
10-20	5	15	75
20-30	12	25	300
30-40	35	35	1225
40-50	24	45	1080
50-60	16	55	880

$n = \Sigma f = 100$ $\Sigma fx = 3600$

$$\therefore \quad \textbf{Mean} = \frac{\sum fx}{n}$$

$$= \frac{3600}{100} = \mathbf{36} \quad \textbf{Ans.}$$

***Question* 7 :**

(a) Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed. [3]

Solution :

$\because$ Volume of the cone = Sum of volumes of the two melted spheres

$$\Rightarrow \quad \frac{1}{3}\pi(r)^2 \times 8 = \frac{4}{3}\pi \times (2)^3 + \frac{4}{3}\pi \times (4)^3$$

$$\Rightarrow \quad 8r^2 = 32 + 256$$

$$\Rightarrow \quad 8r^2 = 288 \quad i.e. \quad r^2 = \frac{288}{8} = 36$$

$\Rightarrow$ $r = 6$ *i.e.* **The radius of the cone = 6 cm** **Ans.**

(b) Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leave the same remainder when divided by $x + 3$. [3]

Solution :

$x + 3 = 0 \Rightarrow x = -3$

Given : remainder when $ax^3 + 3x^2 - 9$ is divided by $x + 3$

= remainder when $2x^3 + 4x + a$ is divided by $x + 3$

$$\Rightarrow \quad a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$$

$$\Rightarrow \quad -27a + 27 - 9 = -54 - 12 + a$$

$\Rightarrow$ $-28a = -84$ *i.e.* $\boldsymbol{a = 3}$ **Ans.**

(c) Prove that $\dfrac{\sin\theta}{1-\cot\theta} + \dfrac{\cos\theta}{1-\tan\theta} = \cos\theta + \sin\theta$. [4]

Solution :

$$\textbf{LHS} = \frac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1-\dfrac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} + \frac{\cos^2\theta}{\cos\theta - \sin\theta}$$

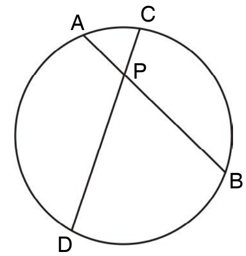
$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} - \frac{\cos^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta - \cos\theta} = \sin\theta + \cos\theta = \textbf{RHS}$$ **Hence Proved.**

***Question* 8 :**

(a) AB and CD are two chords of a circle intersecting at P. Prove that AP × PB = CP × PD. [3]



Solution :

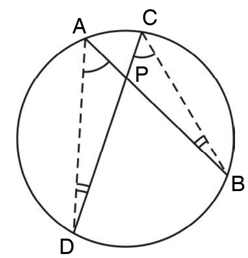
Join AD and BC.

In triangles APD and CPB,

$\angle A = \angle C$ [Angles of the same segment]

$\angle D = \angle B$ [Angles of the same segment]

$\Rightarrow$ $\Delta APD \sim \Delta CPB$ [By A.A. postulate]



$\Rightarrow$ $\dfrac{AP}{CP} = \dfrac{PD}{PB}$ $\Rightarrow$ **AP × PB = CP × PD** **Hence Proved.**

(b) A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is :

(i) a green ball.

(ii) a white or a red ball.

(iii) neither a green ball nor a white ball. [3]

Solution :

Total possible outcomes = 5 + 6 + 9 = 20

(i) $\because$ No. of favourable cases = No. of green balls = 9

$\therefore$ **Probability of drawing a green ball** = $\mathbf{\dfrac{9}{20}}$ **Ans.**

(ii) $\because$ No. of favourable cases = 5 + 6 = 11

$\therefore$ **Required probability** = $\mathbf{\dfrac{11}{20}}$ **Ans.**

(iii) Ball drawn is neither a green nor a white ball

$\Rightarrow$ Ball drawn is red

So the no. of favourable cases = no. of red balls = 6

$\therefore$ **Required probability** = $\dfrac{6}{20} = \mathbf{\dfrac{3}{10}}$ **Ans.**

(c) Rohit invested ₹ 9,600 on ₹ 100 shares at ₹ 20 premium paying 8% dividend. Rohit sold the shares when the price rose to ₹ 160. He invested the proceeds (excluding dividend) in 10% ₹ 50 shares at ₹ 40. Find the :

(i) original number of shares.

(ii) sale proceeds.

(iii) new number of shares.

(iv) change in the two dividends. [4]

Solution :

(i) $\because$ Investment = ₹ 9,600

and, market value of each share = ₹ 100 + ₹ 20 = ₹ 120

$\therefore$ **Original number of shares** = ₹ $\dfrac{9,600}{120}$ = **80** **Ans.**

(ii) $\because$ Each share is sold for ₹ 160

$\therefore$ **Sale-proceeds** = 80 × ₹ 160 = ₹ **12,800** **Ans.**

(iii) Now, investment = ₹ 12,800

and, market value of each share = ₹ 40

$\therefore$ **New number of shares** = ₹ $\dfrac{12,800}{40}$ = **320** **Ans.**

(iv) **Dividend in the 1st case :**

= No. of shares × rate of dividend × N.V. of each share

= 80 × 8% × ₹ 100 = ₹ 640

Dividend in the 2nd case :

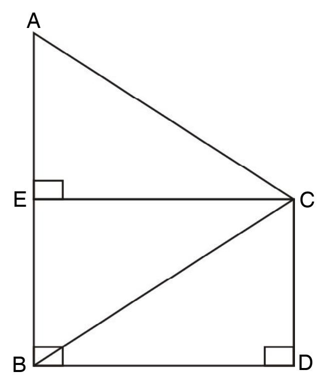
= 320 × 10% × ₹ 50 = ₹ 1,600

$\therefore$ **Change (increase) in two dividends** = ₹ 1,600 – ₹ 640 = ₹ **960** **Ans.**

***Question* 9 :**

(a) The horizontal distance between two towers is 120 m. The angle of elevation of the top and angle of depression of the bottom of the first tower as observed from the top of the second tower is 30° and 24° respectively.

Find the height of the two towers. Give your answer correct to 3 significant figures. [4]



Solution :

In ΔBCD, $\tan 24° = \dfrac{CD}{120\text{ m}} \Rightarrow 0{\cdot}4452 = \dfrac{CD}{120\text{ m}}$

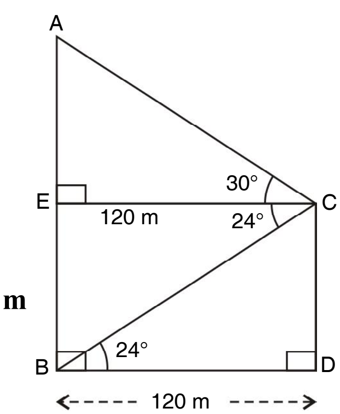
$\Rightarrow$ CD = 120 × 0·4452 m

= 53·424 m = EB

In ΔAEC, $\tan 30° = \dfrac{AE}{120\text{ m}} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{AE}{120\text{ m}}$

$\Rightarrow$ **AE** = $\dfrac{120}{\sqrt{3}}$ m = $40\sqrt{3}$ m

= 40 × 1·732 m = **69·28 m**



$\Rightarrow$ **Height of tower AB** = AE + EB

= 69·28 m + 53·424 m

= 122·704 m = **123 m** (correct to 3 significant figures)

Ans.

And, **height of tower CD** = 53·424 m

= **53·4 m** (correct to 3 significant figures) **Ans.**

(b) The weight of 50 workers is given below :

Weight in Kg	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**	**100-110**	**110-120**
No. of workers	**4**	**7**	**11**	**14**	**6**	**5**	**3**

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2 cm = 5 workers along the other axis. Use the ogive drawn to estimate the following :

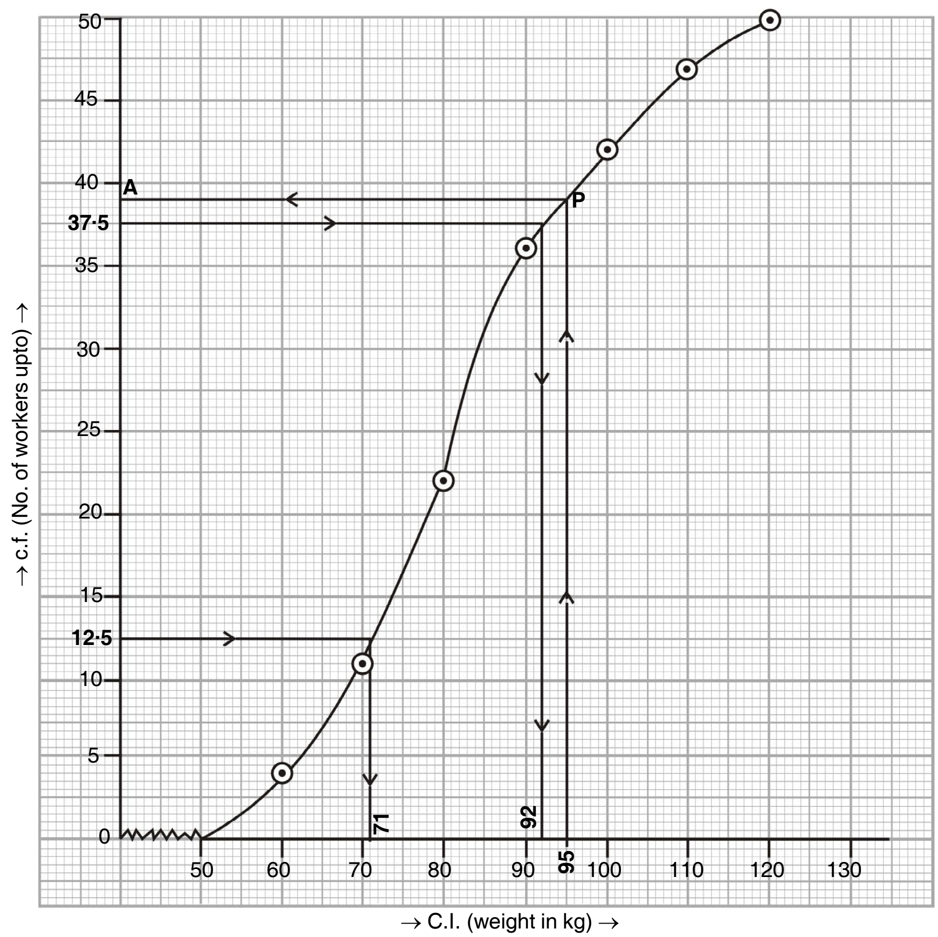
(i) the upper and lower quartiles.

(ii) if weighing 95 Kg and above is considered overweight, find the number of workers who are overweight. [6]

Solution :

Weight in Kg	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of workers (f)	4	7	11	14	6	5	3
c.f.	4	11	22	36	42	47	50

Plot the points (60, 4), (70, 11), (80, 22), (90, 36), (100, 42), (110, 47) and (120, 50). Then draw a free-hand curve (ogive) as shown below :



$\therefore \quad N = 50$

(i) **Upper quartile** $= \left(\dfrac{3 \times N}{4}\right)^{th}$ term

$= \left(\dfrac{3 \times 50}{4}\right)^{th}$ term $= 37{\cdot}5^{th}$ term $=$ **92 kg** **Ans.**

Lower quartile $= \left(\dfrac{N}{4}\right)^{th}$ term

$= \left(\dfrac{50}{4}\right)^{th}$ term $= 12{\cdot}5^{th}$ term $=$ **71 kg** **Ans.**

(ii) Through 95 kg mark, draw a vertical line that meets graph at point P. Through point P, draw a horizontal line which meets axis for *c.f.* at point A and A = 39.

$\Rightarrow$ Weights of 39 workers are 95 kg or below it.

$\therefore$ **Number of workers who are overweight** = 50 – 39 = **11** **Ans.**

***Question* 10 :**

(a) A wholesaler buys a TV from the manufacturer for ₹ 25,000. He marks the price of the TV 20% above his cost price and sells it to a retailer at 10% discount on the marked price. If the rate of VAT is 8%, find the :

(i) marked price.

(ii) retailer's cost price inclusive of tax.

(iii) VAT paid by the wholesaler. **[3]**

Solution :

(i) **Marked price** = ₹ 25,000 + 20% of ₹ 25,000

= ₹ 25,000 + ₹ 5,000 = **₹ 30,000** **Ans.**

(ii) **For retailer :**

C.P. = Marked price – Discount

= ₹ 30,000 – 10% of ₹ 30,000

= ₹ 30,000 – ₹ 3,000 = ₹ 27,000

Tax on it = 8% of ₹ 27,000

$= \dfrac{8}{100} \times$ ₹ 27,000 = ₹ 2,160

$\therefore$ **C.P. inclusive tax** = ₹ 27,000 + ₹ 2,160 = **₹ 29,160** **Ans.**

(iii) **VAT paid by wholesaler**

= Tax on S.P. – Tax on C.P.

= 8% of ₹ 27,000 – 8% of ₹ 25,000

= 8% of ₹ 2,000 = $\dfrac{8}{100} \times$ ₹ 2,000 = **₹ 160** **Ans.**

(b) If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ **and** $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$.

Find AB – 5C. **[3]**

Solution :

$$AB - 5C = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

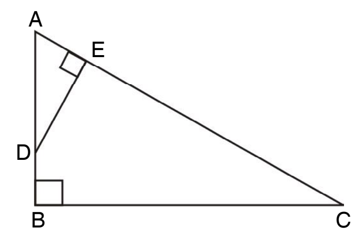
$$= \begin{bmatrix} 35-5 & 27+25 \\ 20+20 & 16-30 \end{bmatrix} = \begin{bmatrix} \mathbf{30} & \mathbf{52} \\ \mathbf{40} & \mathbf{-14} \end{bmatrix}$$ **Ans.**

(c) ABC is a right angled triangle with $\angle ABC = 90°$. D is any point on AB and DE is perpendicular to AC. Prove that :

(i) $\Delta ADE \sim \Delta ACB$.

(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.

(iii) Find, area of ΔADE : area of quadrilateral BCED. [4]

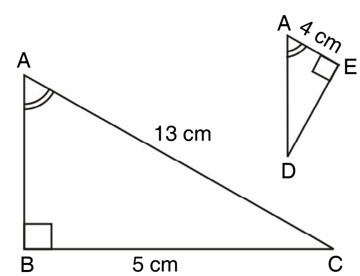


Solution :

(i) $\because \quad \angle ABC = \angle AED = 90°$

$\angle BAC = \angle DAE$ (Common)

$\Rightarrow$ **$\Delta ADE \sim \Delta ACB$** (By A.A. postulate)



(ii) In ΔABC,

$\Rightarrow \quad AB^2 + BC^2 = AC^2$ [Pythagoras theorem]

$\Rightarrow \quad AB^2 + 5^2 = 13^2$

$\Rightarrow \quad AB^2 = 169 - 25 = 144$ and $AB = 12$ cm

$\because \Delta ADE \sim \Delta ACB$

$$\Rightarrow \quad \frac{AD}{AC} = \frac{DE}{BC} = \frac{AE}{AB}$$

$$\Rightarrow \quad \frac{AD}{13} = \frac{DE}{5} = \frac{4}{12}$$

$$\Rightarrow \quad \frac{AD}{13} = \frac{4}{12} \text{ and } \frac{DE}{5} = \frac{4}{12}$$

$$\Rightarrow \quad AD = \frac{4 \times 13}{12} \text{ cm and } DE = \frac{5 \times 4}{12} \text{ cm}$$

$\Rightarrow \quad$ **AD = $4\frac{1}{3}$ cm** and **DE = $1\frac{2}{3}$ cm** **Ans.**

(iii) $\because$ $\Delta ADE \sim \Delta ACB$

$$\Rightarrow \quad \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ACB} = \frac{AE^2}{AB^2} = \frac{4^2}{12^2} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta ACB) - \text{Area}(\Delta ADE)} = \frac{1}{9-1} = \frac{1}{8}$$

$$\Rightarrow \quad \frac{\text{Area}(\Delta ADE)}{\text{Area (quadrilateral BCED)}} = \frac{1}{8}$$

i.e. **Ar. (ΔADE) : Ar. (quadrilateral BCED) = 1 : 8** **Ans.**

***Question* 11 :**

(a) Sum of two natural numbers is 8 and the difference of their reciprocal is $\frac{2}{15}$. Find the numbers. **[3]**

Solution :

Let the natural numbers be x and $8 - x$

$\Rightarrow \quad \frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$ *i.e.* $\frac{8-x-x}{x(8-x)} = \frac{2}{15}$

$\Rightarrow \quad 2(8x - x^2) = 15(8 - 2x)$ *i.e.* $16x - 2x^2 = 120 - 30x$

$\Rightarrow \quad 2x^2 - 46x + 120 = 0$ *i.e.* $x^2 - 23x + 60 = 0$

$\Rightarrow \quad x^2 - 20x - 3x + 60 = 0$ *i.e.* $x(x - 20) - 3(x - 20) = 0$

$\Rightarrow \quad (x - 20)(x - 3) = 0$ *i.e.* $x - 20 = 0$ or $x - 3 = 0$

$\Rightarrow \quad x = 20$ or $x = 3$

Reject $x = 20$ as the sum of natural numbers is 8.

$\therefore \quad x = 3$ and $8 - x = 8 - 3 = 5$

$\Rightarrow$ **Required natural numbers are 3 and 5.** **Ans.**

(b) Given $\frac{x^3 + 12x}{6x^2 + 8} = \frac{y^3 + 27y}{9y^2 + 27}$. Using componendo and dividendo, find $x : y$. **[3]**

Solution :

Applying componendo and dividendo, we get :

$$\frac{x^3 + 12x + 6x^2 + 8}{x^3 + 12x - 6x^2 - 8} = \frac{y^3 + 27y + 9y^2 + 27}{y^3 + 27y - 9y^2 - 27}$$

$\Rightarrow \quad \frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$ *i.e.* $\frac{x+2}{x-2} = \frac{y+3}{y-3}$

$\Rightarrow \quad \frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$ [Applying componendo and dividendo]

$\Rightarrow \quad \frac{2x}{4} = \frac{2y}{6}$ *i.e.* $\frac{x}{2} = \frac{y}{3}$

$\Rightarrow \quad 3x = 2y$ and $\frac{x}{y} = \frac{2}{3}$ *i.e.* $\boldsymbol{x : y = 2 : 3}$ **Ans.**

(c) **Construct a triangle ABC with AB = 5·5 cm, AC = 6 cm and ∠BAC = 105°. Hence :**

  (i) **Construct the locus of points equidistant from BA and BC.**

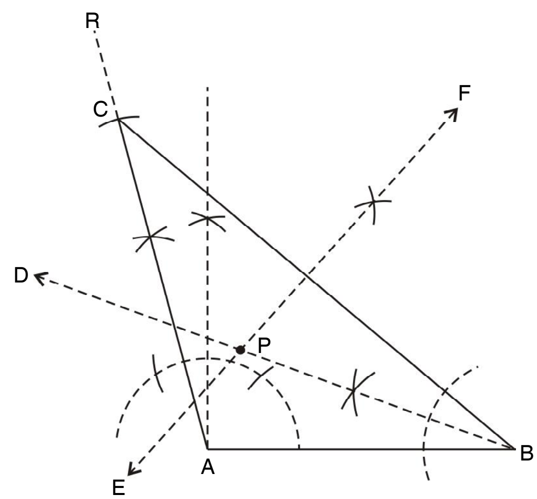
  (ii) **Construct the locus of points equidistant from B and C.**

  (iii) **Mark the point which satisfies the above two loci as P. Measure and write the length of PC.** **[4]**

Solution :

Steps to construct ΔABC.

1. Draw AB = 5·5 cm.
2. Construct angle BAR = 105°.
3. From AR, cut AC = 6 cm and complete the triangle ABC.



(i) Draw BD, the bisector of angle ABC, which is the locus of points equidistant from BA and BC.

(ii) Draw EF, the perpendicular bisector of BC, which is the locus of points equidistant from B and C.

(iii) BD and EF intersect each other at point P.

∴ **P satisfies the above two loci.**

Also, **PC = 4·8 cm** (app.) **Ans.**